Chapter 13 - Simple Harmonic Motion (SHM)

The acceleration for an object in SHM is directly proportional to the displacement from the equilibrium position and is in the opposite direction to the displacement.

Period of a Mass on Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where T is the period of oscillation (s)
m is the mass (kg)
k is the spring constant ($\frac{N}{m}$)

displacement from equilibrium

Recall: $F_a = kx$ (Hooke's Law $\rightarrow$ $F = -kx$ for restoring force)
$E_e = \frac{1}{2}kx^2$ (Elastic Potential Energy)
$E_k = \frac{1}{2}mv^2$ (Kinetic Energy)

Period of a Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

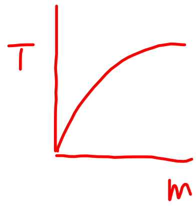
Where T is the period of the pendulum (s)
l is the length of the pendulum (m)
g is the acceleration due to gravity (m/s^2)

Recall: $E_g = mgh$ (gravitational potential energy)
$E_k = \frac{1}{2}mv^2$ (Kinetic energy)

Determine the spring constant graphically:

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow T \propto \sqrt{m}$$

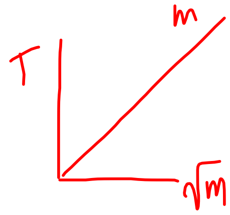
a graph of T vs m would be a root curve



$$T^2 = \frac{4\pi^2}{k} m$$

$T^2 \propto m$

T vs $\sqrt{m}$ would be linear



A graph of T^2 vs m will be linear with a slope of $\frac{4\pi^2}{k}$

← slope = $\frac{4\pi^2}{k}$

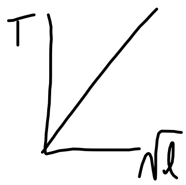
Determine the acceleration due to gravity graphically:

$$T = 2\pi\sqrt{\frac{l}{g}} \rightarrow T \propto \sqrt{l}$$

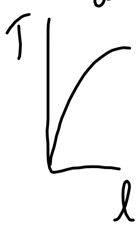
$$T^2 = \frac{4\pi^2}{g} l$$

$T^2 \propto l$

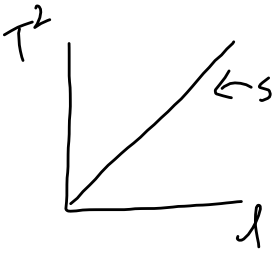
A graph of T vs $\sqrt{l}$ will be linear



root curve.



A graph of T^2 vs l will be linear with a slope of $\frac{4\pi^2}{g}$



← slope = $\frac{4\pi^2}{g}$

MP|606

$x = 12.0cm$

$m = 125g$

20.0cycles in 15.5s

a) $T = ?$

b) $k = ?$

c) $E_{TOT} = ?$

d) $v_{max} = ?$

e) $v = ?$ at 10.0cm

a) $T = \frac{15.5s}{20.0\,cycles}$

$T = 0.775\ s$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = \frac{4\pi^2 m}{k}$

$k = \frac{4\pi^2 m}{T^2}$

$k = \frac{4\pi^2(0.125kg)}{(0.775s)^2}$

$k = 8.22\ N/m$

c) $E_{TOT} = \frac{1}{2}kx^2$ ← elastic potential energy

$E_{TOT} = \frac{1}{2}\left(8.22\frac{N}{m}\right)(0.120m)^2$

$E_{TOT} = 0.0592J$

d) max speed => max E_k => $E_e = 0$ => at equilibrium. $(x=0)$

$E_{TOTAL} = E_k + \cancel{E_e}^{\,0}$

$0.0592J = \frac{1}{2}(0.125kg)v^2$

$v = 0.973m/s$

e) At 10.0 cm from equilibrium:

$E_{TOT} = E_e + E_k$

$0.0592J = \frac{1}{2}\left(8.22\frac{N}{m}\right)(0.100m)^2 + \frac{1}{2}(0.125kg)v^2$

$0.0592J = 0.0411\ J + \frac{1}{2}(0.125kg)v^2$

$0.0181J = \frac{1}{2}(0.125kg)v^2$

$v = 0.538\ m/s$

TO DO

PP|608 → mass on spring

MP|613 + PP|614 → pendulum